



Mid Term Examination, Spring 2023
 Department of Electrical and Electronic Engineering (EEE)
 Level-2, Term-II
 Course Code: MATH 261
 Course Title: Probability and Statistics

Notes:

Time : 1 hr.

Full Marks : 60

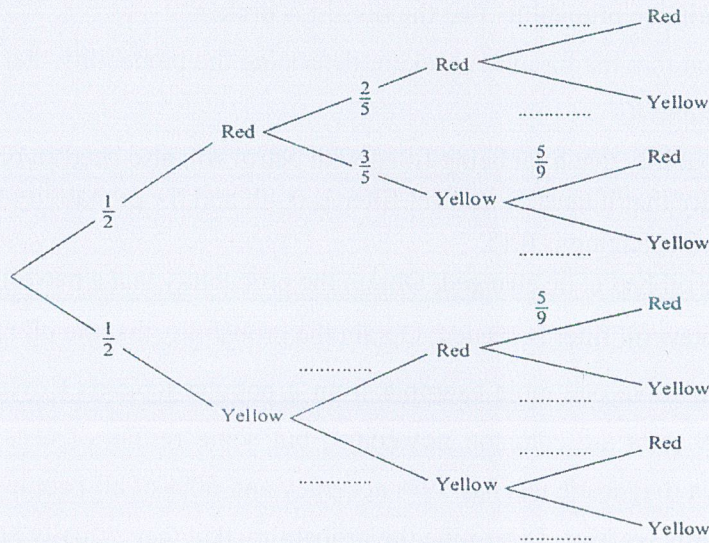
a. Figure on the right of each question indicate marks for respective question.

b. Answer any 02 (Two) of the following 03 (Three) questions.

1. a. Identify the probability of getting a 5 or greater than 10 totals on either of two tosses of a pair of fair dice. 08
- b. In a factory, three machines, J , K and L are used to make biscuits. Machine J makes 25% of the biscuits. Machine K makes 45% of the biscuits. The rest of the biscuits are made by machine L . 14
- It is known that 2% of the biscuits made by machine J are broken, 3% of the biscuits made by machine K are broken and 5% of the biscuits made by machine L are broken.
- i) Illustrate the possible outcomes and associated probabilities using tree diagram when a biscuit is selected at random.
- ii) Calculate the probability that the biscuit is made by machine J and it is not broken.
- iii) Obtain the probability that the biscuit is broken.
- iv) Given that the biscuit is broken, determine the probability that it was not made by machine K .
- c. The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15. 08
- (i) If the oil had to be changed, Obtain the probability that a new oil filter is needed?
- (ii) If a new oil filter is needed, Obtain the probability that the oil has to be changed?
2. a. I want to solve one more example from a popular topic as Covid-19. As you know, Covid-19 tests are common nowadays, but some results of tests are not true. Let's assume; a diagnostic test has 99% accuracy and 60% of all people have Covid-19. If a patient tests positive, Determine the probability that they actually have the disease? 06
- b. Given the following Joint density function of X and
- $$f(x, y) = \frac{6}{5}(x + y^2), \quad 0 < x < 1, 0 < y < 1$$
- $$= 0 \quad \text{elsewhere}$$
- Compute
- i) $f(x|y)$
- ii) $f(x|0.5)$ and
- iii) $P(X < 0.5 | Y = 0.5)$.



- c. A joint probability density function of two random variables X and Y is given by $f(x, y) = 12xy(1 - y)$; $0 < x < 1, 0 < y < 1$. Discuss whether X and Y are independent? 12
3. a. Compute $P(A \cup B)$, when two coins are tossed "A" is the event of "getting two tails" and "B" is the event of "getting head in the second coin". 06
- b. Let X be a random variable with probability function defined by $f(-2) = \frac{1}{10}, f(0) = \frac{2}{10}, f(4) = \frac{4}{10}$ and $f(11) = \frac{3}{10}$. 08
 Estimate
 i) $P[-2 \leq X < 4]$.
 ii) $P[X > 0]$ and
 iii) $P[X \leq 4]$.
- c. The bag P contains 6 balls of which 3 are red and 3 are yellow. The bag Q contains 7 balls of which 4 are red and 3 are yellow. A ball is drawn at random from bag P and placed in bag Q . A second ball is drawn at random from bag P and placed in bag Q . A third ball is then drawn at random from the 9 balls in bag Q . 15
 The event A occurs when 2 balls drawn from bag P are of the same colour.
 The event B occurs when the ball drawn from bag Q is red.
 i) Complete the tree diagram shown below.



- ii) Compute $P(A)$
 iii) Show that $P(B) = \frac{5}{9}$
 iv) Prove that $P(A \cap B) = \frac{2}{9}$
 v) Estimate $P(A \cup B)$.